(1)

B.Sc. II, PHYSICS HONS
OPTICS - Paper-III 3rd

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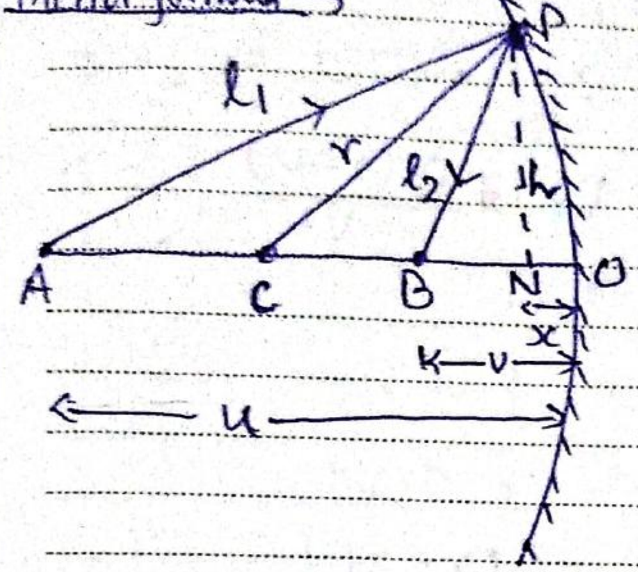
Spherical mirror formula and lens formula using Fermat's principle

Mirror formula :→



Let A is the object at the principal axis of a Concave mirror of radius of curvature r. B is the image of A. The object distance OA = u and the image distance OB = v. Put ON = x and PN = h.

$$\therefore PC^2 = CN^2 + NP^2$$
$$= (OC - ON)^2 + NP^2$$
$$\therefore r^2 = (r-x)^2 + h^2$$
$$= r^2 - 2rx + x^2 + h^2$$
$$\Rightarrow x^2 + h^2 = 2rx \quad \text{---(1)}$$

And $AP^2 = AN^2 + NP^2 = (OA - ON)^2 + NP^2$

$$\therefore l_1^2 = (u-x)^2 + h^2$$
$$= u^2 - 2ux + x^2 + h^2$$
$$= u^2 - 2ux + 2rx \quad \text{(using eqn (1))}$$
$$= u^2 + 2x(r-u)$$
$$\Rightarrow l_1 = \{u^2 + 2x(r-u)\}^{1/2}$$
$$= u\left\{1 + \frac{2x(r-u)}{u^2}\right\}^{1/2}$$
$$\Rightarrow l_1 = u\left\{1 + \frac{1}{2}\cdot\frac{2x(r-u)}{u^2}\right\} \quad \text{(using Binomial Theorem)}$$
$$\Rightarrow l_1 = u\left\{1 + \frac{x(r-u)}{u^2}\right\}$$

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3	4	5	6	7	8	9
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17	18	19	20	21	22	23
24	25	26	27	28	29	30

Similarly, $l_2 = v\left\{1 + \frac{x(r-v)}{v^2}\right\}$

$\therefore$ The optical path APB is given by

$$l = l_1 + l_2$$

For l to be minimum $\frac{dl}{dx}$ must be zero

$$\therefore \frac{d}{dx}\left[u\left\{1+\frac{x(r-u)}{u^2}\right\} + v\left\{1+\frac{x(r-v)}{v^2}\right\}\right] = 0$$

$$\Rightarrow \frac{r-u}{u} + \frac{r-v}{v} = 0$$

$$\Rightarrow \frac{r}{u} - 1 + \frac{r}{v} - 1 = 0$$

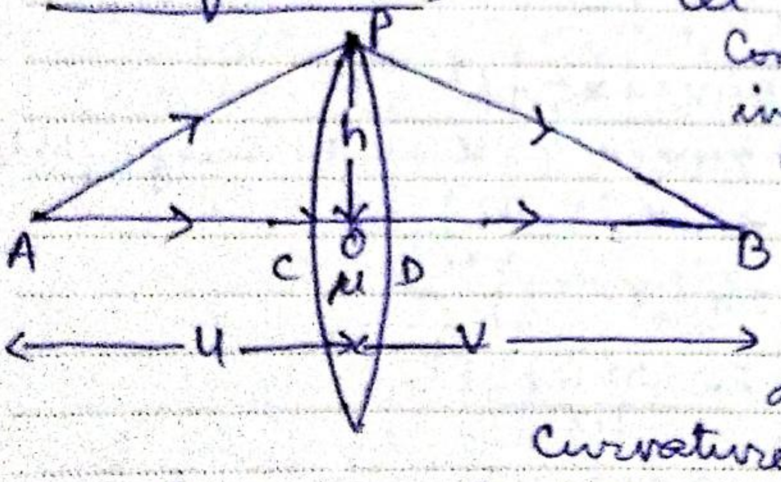
$$\Rightarrow \frac{r}{u} + \frac{r}{v} = 2$$

$$\Rightarrow \frac{1}{u} + \frac{1}{v} = \frac{2}{r} \qquad (r = 2f)$$

$$\therefore \frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

 This is mirror formula.

Lens formula $\rightarrow$



Let us consider a thin convex lens of refractive index μ. Let A is the object and B is its image. Let

u = object distance

v = image distance

and r_1, r_2 = radii of curvature of first and second surface. From the Fermat's principle optical path APB = optical path AOB

$$\Rightarrow AP + PB = AB - CD + \mu \cdot CD$$

$$= AB + CD(\mu - 1) \quad \text{—— (1)}$$

(3)

For first Surface,

$PO^2 = CO\cdot(2r_1 - CO) = 2r_1\cdot CO - CO^2$

$\therefore h^2 = 2r_1\cdot CO$ (neglecting CO^2 for a thin lens)

For Second Surface,

$h^2 = 2r_2\cdot DO$

$\therefore CO = \frac{h^2}{2r_1}$ and $DO = \frac{h^2}{2r_2}$

$\therefore CD = CO + OD = \frac{h^2}{2}\left(\frac{1}{r_1} + \frac{1}{r_2}\right)$

Now, $AP = (AO^2 + OP^2)^{1/2} = (u^2 + h^2)^{1/2}$

$= u\left(1 + \frac{1}{2}\cdot\frac{h^2}{u^2}\right)$

$BP = (BO^2 + OP^2)^{1/2} = (v^2 + h^2)^{1/2}$

$= v\left(1 + \frac{1}{2}\cdot\frac{h^2}{v^2}\right)$

$\therefore AP + PB = (u+v) + \frac{h^2}{2}\left(\frac{1}{u} + \frac{1}{v}\right)$

$= AB + \frac{h^2}{2}\left(\frac{1}{u} + \frac{1}{v}\right)$ —— (2)

from (1) and (2)

$AB + (\mu - 1)\cdot\frac{h^2}{2}\left(\frac{1}{r_1} + \frac{1}{r_2}\right) = AB + \frac{h^2}{2}\left(\frac{1}{u} + \frac{1}{v}\right)$

$\therefore \frac{1}{v} + \frac{1}{u} = (\mu - 1)\left(\frac{1}{r_1} + \frac{1}{r_2}\right)$ —— (3)

when $u = \infty$, $v = f$ and using Sign convention ($r_1 = +ve$ & $r_2 = -ve$), we have

$\frac{1}{f} = (\mu - 1)\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$

and $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$ (Putting the value of (3) we get it)

This is lens formula.

—x—x—